Alternate approaches to modeling the financial impacts of climate change in DICE 2016

Math 695

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# Abstract

The Dynamic Integrated Climate and Economy Model (DICE) is an Integrated Assessment Model (IAM) currently used by the World Economic Forum and other global bodies to calculate the long-term impact of climate change on the economy (I2AMParis, 2016). At present, DICE is a cooperative model and suggests the impact of climate change attributed to emissions on the global economy will be noticeable (Nordhaus, 2018), but not as catastrophic as is believed by climate scientists (Ewing et. al., 2010). To better reflect ecological findings and economic relationships, alternate models derived from DICE are proposed to incorporate the negative effects these emissions may have on global climate change, and the resulting effects on economic output (Gillingham, 2019). The alternate models implement more realistic competitive relationships between economic output and emissions via predator-prey, shared resource, and legal monopoly competition. In addition to these alternate discrete models, continuous approximations of the proposed predator-prey and shared resource competition models comprised of differential equations are proposed and examined.

# Introduction

The ideas motivating the development of math and science can be well encompassed by a single question: *Can we explain what happens in the world around us?* Mathematicians and scientists of old struggled with this question in their pursuit of knowledge. One such mathematician was Isaac Newton, who was enthralled by the teachings of the ancient philosophers, Aristotle and Plato, and their contemporaries, Descartes and Gassendi, in his time at Trinity College Cambridge (Westfall, 2023). It is thought to be their writings which inspired Newton to pursue mechanics and physics, culminating in his laws of motion, definition of gravity, and the creation of calculus. In short, Newton’s intuition was the catalyst of modern mathematics.

Since the time of Newton, science and mathematics have evolved to such a point that many natural phenomena are well studied. One area of study in the modern era is the impact climate change will have on humanity. Since 1820 when atmospheric measures of carbon dioxide (CO2) were first recorded, the density of carbon in the atmosphere has increased from 182 to 412 parts per million (Stein, 2022). As a consequence of this increase, it is widely accepted by those studying the climate, that an average global temperature increase of 1.5 °C by 2050 is the minimum possible outcome, with an increase of 2º being most likely (United Nations, “The Paris Agreement”). While there is much room for further study, climate scientists, ecologists, and biologists of many kinds have used this knowledge to approximate the future state of the world. There is much consensus on the effect climate change will have on sea levels (Tamaki et. al. 2019), agriculture, and human displacement (Ritchie and Roser, Feb. 2021). One area that demands further work is economics.

Studying the economic impacts of climate change has been an active area of research for several decades. Much of the early work was empirical, that is using available weather and economic data to describe how climate change had thus-far impacted certain economic sectors (Kolstad and Moore, 2020). The field has since evolved to attempt to be predictive through the use of integrated assessment models, or IAMs. Such models aim to provide far-reaching insights into environmental change and economic development via quantitative depictions of climate properties (such as average global temperature, sea level, or change in biodiversity) and economic systems (Nordhaus, Apr. 2017). These complex financial models are used to depict economic outcomes at a number of scales, from predicting the performance of a single stock, to optimizing the economies of scale of mega-corporations (Nordhaus, Apr. 2017). The predictions generated by IAMs are thus used by governments and corporations alike to guide investments and prepare for the future.

# DICE 2016

Originally developed by Nobel laureate and professor of economics at Yale University, William Nordhaus, the Dynamic Integrated Climate and the Economy model (DICE) is an IAM of such notoriety that its calculations and predictions have been used to guide economic policy at the global scale. DICE, and its regional variant RICE, were first introduced in 1992 as one of the earliest IAMs for climate change (Nordhaus, Apr. 2017). The DICE family of models have since undergone much development to account for more recent economic, scientific, and environmental data. Many of these changes made have been to the economic sector, as the predictions of economic growth were much lower than achieved and advancements in technology have had a more significant economic impact than first projected.

The DICE model consists of two overarching modules, one for modeling the economy and the other for climate. The economic module accounts for factors driving economic growth like: labor; population; capital; technology; and their respective greenhouse gas emissions, and attempts to define the global economy deterministically despite the underlying unpredictability of global finance. In particular, the economic module operates under the assumption that the goal of the global economy is to optimize its own welfare rather than generate the greatest possible capital. This distinction defines the global economy as cooperative, in contrast to the standard competitive definition assumed by economists (Monopolistic Competition, Oligopoly, and Monopoly, 2016).

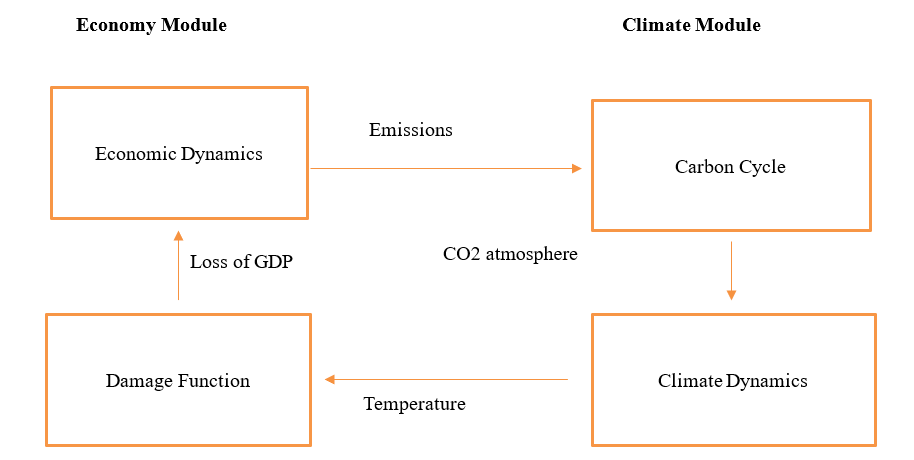


Figure 1: Module Layout of DICE Model (I2AMParis, 2016)

The intent of the model is to optimize a social welfare function, W, which in principle is to calculate the outcome of a market with constraints over time. The welfare function is defined as a discounted sum of utility, U(t), and is comprised of underlying functions representing global consumption measured in trillions of United States dollars (USD), C(t), population in billions of people, L(t), a welfare factor, , and parameter . The parameters and are defined as the generational aversion to inequality, and the generational discount rate respectively and are assumed to be 1.45 and 0.3 respectively (Nordhaus and Sztorc, 2013).

Equation 1: Welfare Function

Equation 2: Utility Function

To quantify utility, global consumption is calculated as the difference between net output and investment, C(t) = Q(t) – I(t). Nordhaus defines net output, Q(t), as a function of gross output, Y(t), both measured in trillions of USD. In particular, Q(t) denotes output as it is affected by climate damages and carbon abatement, and Y(t) is a Cobbs-Douglas function of capital in trillions of USD, K(t); labor (which is exogeneous and assumed to be population in billions of people), N(t); and a technology productivity factor, A(t), which is again exogeneous. The elasticity of the Cobbs-Douglas function is given by , which is between 0 and 1, and denotes the rate of substitution between capital and technology (Nordhaus and Sztorc, 2013). is initially assumed to be 0.3. Note that an alternative definition for Q(t) is the sum of consumption, C(t), and gross investment, I(t) (Cooke, 2012). Capital in the next time period, K(t+1), is the sum of the depreciated capital of the previous period at a rate of , and investment. It is important to note that the rate of change of capital is dependent only on the current values of the output function, and there are no other opportunities for growth.

Equation 3: Net Output Function

Equation 4: Gross Output Function

Equation 5: Capital Function

Economic damages and carbon abatement cost are defined by (t) and (t) respectively, and the damage function, D(t), quantifies the assumed economic impacts associated with changes in global temperature. Since the DICE model represents climate impacts on the economy at a global scale, Nordhaus chose to define the damage function as a quadratic in terms of the global average temperature of the atmosphere, to reflect how temperature increases can lead to positive feedbacks in the climate system leading to possible accelerations in climate damages (Kellett et. al., 2019). The parameters and default to 0.00267 and 2 respectively which yield an economic damage of 3.75% of gross output with a temperature increase of 3ºC (Tol, 2009 & Nordhaus and Sztorc, 2013), but can be altered to balance the damage function if a specific damage ratio is chosen. Lastly, abatement cost is a function of emissions reductions, (t); the number of technological alternatives at the time for abatement given by, ; and the degree of nonlinearity for deeper cuts in emissions, ; is assumed to be 2.8 under the assumption there exists a maximum cost beyond which any abatement is more expensive than the cost of carbon it offsets (Tamaki et. al., 2019).

Equation 6: Overall Damage Function

Equation 7: Temperature Induced Damage Function

Equation 8: Abatement Function

The climate module consists of equations for geophysical systems and their interactions with various economic sectors. Key relationships modeled in the climate module are: a radiative forcing equation to quantify the impact increases in carbon emissions can have on atmospheric temperature; equations for calculating the reserves of carbon in the atmosphere, upper oceans, and lower oceans (the global carbon cycle); and an emissions-output relationship (I2AMParis, 2016).

Uncontrolled CO2 emissions are given by a measure of carbon intensity given by where , weighted by gross output Y(t). Using this baseline, Nordhaus defines total CO2 emissions, E(t), as the sum of uncontrolled emissions and additional land-use carbon emissions, Eland(t), reduced by the carbon mitigations of emission reduction. To link the net emissions function with the natural processes of the earth, geophysical equations are used to define the relationships between emissions and the carbon cycle, radiative forcings, and overarching climate change. Mj(t) defines the amount of carbon between 3 reservoirs: atmosphere, MAT(t); upper oceans, MUP(t); and lower oceans, MLO(t). Weighting the flow of carbon between these reservoirs are the parameters , with i denoting the biosphere the carbon is transitioning from, and j denoting the biosphere the carbon is transitioning to. These parameters are constant and set by Nordhaus (Tamaki et. al., 2019).

Equation 9: Emissions Function

Equation 10: Carbon Cycle Functions

Radiative forcing is the process describing the absorption of solar radiation by the Earth’s atmosphere (Lightfoot and Orval, 2018). The DICE model defines F(t) as the net change in radiative forcings due to atmospheric conditions, (t), and exogenous forcing *FEX*(t). Global warming is influenced by forcings as described in the 2-stage global climate model, *TAT*(t) and *TLO*(t), of mean global upper atmospheric and lower ocean temperatures in Celsius respectively. The parameters are constants representing the weight radiative forcing has on the temperature change and are explained in depth in the manual for the DICE model (Nordhaus and Sztorc, 2013).

Equation 11: Radiative Forcing Function

Equation 12: Upper Atmospheric Temperature Function

Equation 13: Lower Atmospheric Temperature Function

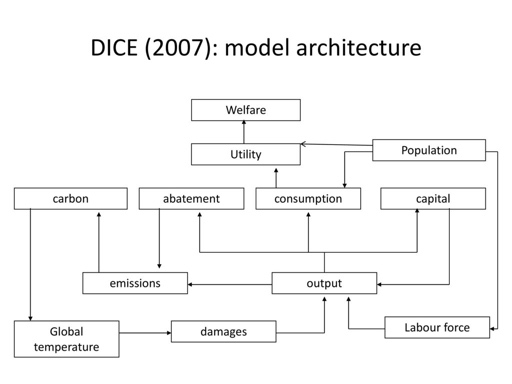


Figure 2: Flow Diagram of DICE Equations (Nordhaus, Apr. 2017)

To demonstrate the interconnectedness of the various functions of the model, simple substitutions yield the following:

Equation 14: Integrated Welfare Function

with climate associated damages given by:

Equation 15: Integrated Damage Function

and temperature changes associated to climate change given by:

Equation 16: Integrated Radiative Forcing Function

along with equation 13.

The overarching modules are explicitly connected via the damage and emissions functions. As stated, the damage function is dependent on the change in temperature, which is a result of the climate module; and the emissions function calculates emissions as a ratio of gross output, which is a result of the economic module. There are no other direct relationships between the two modules, suggesting that in DICE, the interactions between the global economy and the impact of emissions on the climate are limited to an impact of gross output alone.

# Quantifying the Economic Impacts of Climate Change

The ecological impact emissions have on climate change are widely agreed upon by climate scientists: emissions induced climate change is associated with more frequent and more destructive natural disasters. While the long-term effect of these events is presently unknown, recent events indicate that future natural disasters may cause even greater economic damages than modern events (Mizutori, 2018). To combat even further damages, economists and policy makers have worked alongside climate experts to set achievable emissions goals for governments and companies alike (United Nations, “The Paris Agreement”). Underpinning these policies is the uncertainty of the cost of implementing proposed climate policies and their associated impact.

Estimating the financial impact carbon reduction policies can have on an economy is fundamentally a cost-benefit analysis (Association of Environmental and Resource Economists, 2014). To that end, determining the costs of implementing a policy tend to be more clear than quantifying the benefit. To calculate short-term costs of carbon mitigation strategies, economists estimate the costs of implementing the strategy and divide by the net reduction in tons of emissions (Nordhaus, Feb. 2017). For example, if a $1000 government rebate exists for purchasing an electric vehicle and using this vehicle decreases carbon emissions by .1 tons over the lifespan of the vehicle, the short-term mitigation cost is $10000 per ton. Though a simple calculation, the short-term mitigation cost is most commonly used when comparing mitigation policies. That said, short-term mitigation costs are rarely accurate at scale (Nordhaus, Apr. 2018). More often than not, short-term policies are intertwined to the degree that calculating their individual mitigation costs are improbable. Additionally short-term mitigation strategies are greatly influenced by location and existing infrastructure, and are thus infrequently used as a determining factor in deciding to undertake a mitigation strategy (Nordhaus, Feb. 2017).

An alternate approach to calculating the benefit of a mitigation strategies to determine how much damage carbon does to gross output. One such method of determining this damage is to calculate the Social Cost of Carbon (SCC). SCC represents the economic cost caused by an additional ton of carbon dioxide emissions or its equivalent. Calculating an accurate figure for SCC is one of the most important factors of Climate-Economy IAMS such as DICE (Association of Environmental and Resource Economists, 2014). With a reasonable value for SCC, financial impacts of climate policies can be more easily predicted – and thus – prepared for. SCC attempts to remedy the issues of short-term mitigation costs by calculating a marginal unit impact of emissions with respect to the rate of consumption.

Equation 17: Function for Social Cost of Carbon

By defining the cost of carbon with respect to consumption, SCC is a more widely applicable figure in estimating the benefits of mitigation strategies. Interestingly, SCC demonstrates that improving the climate comes at a cost to the economy, as lower consumption leads to lower output (Economic Activity Indicators, International Monetary Fund). That said, such a relationship may not always be the case. In the event of technological advancements, mitigation strategies may be developed that are more financially beneficial than their counterparts. For example, advancements in battery technology could lower the price of storing electricity to the point that wide-scale solar farms are more profitable and efficient than fossil fuel based electricity generation (Ring, 2022). To that end, estimates of SCC can vary greatly and are heavily reliant on assumptions of technological advancement and economic output.

# Limitations of DICE 2016

In contrast to the ecological impacts, the economic impacts of emissions induced climate change are widely debated. The International Monetary Fund has published multiple articles questioning the impacts climate change can have on the economy. Many believe that the long term impacts of climate change will be beneficial to the global economy, citing increases in agriculture and the development of technology to combat the more negative aspects of climate change (United States Environmental Protection Agency, 2022). In the DICE model the negative impacts of climate change on total output, Q(t), are quantified with the damage and abatement functions. Describing the output directly with changes in temperature and abatement efforts displays an interesting relationship. First consider the equivalent definition of the integrated damage function:

then incorporating the remaining variables from Equation 4 yields:

Equation 18: Integrated Net Output Function

To explore the long term ramifications of this relationship, suppose Y(t) and are constant and consider the limit of Q(t) as time goes to infinity. Under current environmental conditions, radiative forcings will increase the average temperature of the planet over time as solar radiation is absorbed and contained in the atmosphere by modern elevated levels of greenhouse gasses (Chandler, 2020). Following this behavior to the extreme then yields:

Equation 19: Limit of Net Output Function

Though this behavior is strictly theoretical, it suggests that the economic impacts associated to climate change in the DICE model will decrease as the temperature increases, which directly contrasts with the opinion of climate scientists (Kahn et. al., 2019). For a more plausible calculation, suppose the average global atmospheric temperature increased by 5ºC. Then, with the current average atmospheric temperature of 13.9ºC (Sharp and Stein, 2022), damages are calculated as:

suggesting the damages are roughly 1/700th of gross output. Such behavior is unrealistic as a temperature increase of 4ºC is estimated to raise sea levels between 0.5 and 1 meter (GreenFacts Scientific Board, 2013), which on its own would be catastrophic to global economy.

This observation is further supported by examining the capital function, K(t+1). By substituting the equation for gross output, Q(t) into the capital function and replacing the difference equation with a differential equation, the model reduces to a first-order linear differential equation:

Equation 20: Differential Capital Function

with implicit solution:

Equation 21: Solution of Differential Capital Function

Observe that the initial value of capital, K(0), is the solution at t = 0 as the integral is undefined. Substituting in that value for c1 provides some insight into the overarching dynamics. Note that the initial value is multiplied with a negative exponential. It follows that this value tends to 0, and so initial capital depreciates as time progresses. With impact of the initial capital decreasing, the only way capital can increase in this system is through the investment term I(t). This demonstrates how the DICE model does not take the impacts of climate change on capital into account, and that other economy influencing variables, like labor L(t) or technological advancements A(t), are not directly influential to the growth of capital.

The existing dynamics implemented in calculating capital in the DICE model seem to contrast the interpretations of the effects of climate change climate scientists propose. To further clarify these dynamics consider the following example from Cooke, 2012: let T(t) = T\* and let K\* denote the equilibrium capital under the temperature change T\* and K0 denote the equilibrium capital with no temperature change. Suppose T\* = 20° C. Such a change in temperature would make life on earth as we know it impossible (Gillingham, 2019). That said the ratio of K\* and K0 is .47, suggesting that with constant population, productivity, abatement effort, and investment only half of all capital stocks are lost to the direct impacts of climate change. Clearly the economic dynamics implemented in the DICE model do not reflect the real-world impacts that the effects of climate change can have on the economy, and structural alternatives to the financial modeling approach are needed.

# Review of Competition Dynamics

Many systems in the natural or social sciences are intertwined to the degree that their outcomes are dependent upon one-another ,and that a change in one system can drastically alter the outcome of its related systems. Such systems are called competitive systems, and they come in many forms. (Lang and Benbow, 2013).

Competition dynamics are well studied and commonly applied to mathematical biology and economics. In general, there are 2 underlying kinds of competition: interspecies and intraspecies. As the names suggest, intraspecies competition defines a relationship where two or more of the same species compete for the same resource, whereas interspecies is competition between different species. The applications in mathematical biology are rather literal and are commonly categorized as: predator-prey, symbiosis, and shared-resource.

In predator-prey competition, one species consumes the other. A more general characterization being that one party gains members from the interaction whereas the other loses members. These dynamics are most commonly seen between carnivores and herbivores, but the relationship holds between herbivores and plants, protozoans and bacteria, and groups of carnivores (Lang and Benbow, 2013).

Symbiotic relationships are characterized by the intentional interaction between species. The most well-known symbiotic relationships are parasitic: where one species gains from interacting with the other without killing and consuming it (Lang and Benbow, 2012). Others can be commensalistic: in which neither species gains nor loses from the interaction; or mutualistic: such that both species benefit from their interaction. The distinction between commensalistic and mutualistic relations is well demonstrated by contrasting the relationships between trees and moss, and hippopotami and oxpeckers. Epiphytes, such as Spanish moss, often grow on the bark of trees as they spread. In doing so there is no benefit nor detriment to the health of the tree, and the moss does not need the tree for its survival; thus their relationship is commensalistic (JoVE Editorial Board, 2016). When compared to the relationship between hippopotamus and the oxpecker – where the oxpecker eats food stuck between the hippo’s teeth – there is clearly benefit for both animals involved, and so their relationship is mutualistic.

Competition for shared resources is likely the most intuitive of the three ecological kinds. It is defined as a relationship between two or more species attempting to control and consume a shared resource (Lang and Benbow, 2013). Such competition is common amongst groups of predators that share a hunting ground, like packs of wolves, or herbivores that eat the same plants, like cows and sheep.

In economics, the applications of ecological competition hold true, but there exist other relationships unseen in nature. Competition in markets is chiefly dependent on the number of sellers available to a buyer, where fewer sellers lead to less competition (Monopolistic Competition, Oligopoly, and Monopoly, 2016). The market with the least competition is called a monopoly. In such a system there is only one seller on the market, and since there are no identical goods on the market provided by other sellers, the seller can charge any price – so long as the buyer can still pay. Most monopolies fall into one of two categories: natural and legal. Natural monopolies are markets where competition is inhibited (usually by the government) because the resource is important to society. Common natural monopolies are utilities like water, electricity transmission, and sewer services (Monopolistic Competition, Oligopoly, and Monopoly, 2016). Legal monopolies arise when a company receives a patent for a product or process, and is thus the exclusive seller. Under such a system, there are penalties for competitors that sell their own version of the patented product.

In a slightly more diversified market, the competition is known as an oligopoly. In such a market, few sellers supply the majority of goods and offer similar products. As such, the sellers can influence the price of their products, but must respect the prices set by other sellers as to not be undercut.

The most common form of market competition is monopolistic competition. In monopolistic markets, a number of sellers offer an array of products that differ slightly but ultimately serve the same purpose. To that end, each seller has a small-scale monopoly over their products, but not over the market as a whole. Monopolistic markets give the consumer the greatest amount of choice while still allowing the seller to control the price of the product to some degree based on the quality, brand, and marketing of the product.

The market with the greatest amount of competition is called a perfect competition market. In these markets many small firms sell near-identical products for the same price. It follows that any product is easily substituted for another, and sellers must unanimously agree upon the price of their products. Markets with perfect competition are almost exclusively theoretical constructs, but there are some markets with near-perfect competition, one such market being basic produce. In this market, the wheat, eggs, milk, and other produce are virtually identical. If a buyer does not like the price of one seller, they can go to another and easily replace products. As such, prices are set almost exclusively by market value rather than by the producer (Wolff, 2022).

# Predator-Prey DICE 2016

The most straight-forward approach in updating the relationship between economy and climate change in DICE is to implement a predator-prey relationship between the two. As described by climate scientists, an unchecked increase in emissions can lead to drastic ecological changes, and even small temperature changes can have profound affects (Kellett et. al., 2019). It follows that with an increase in production – and in turn gross output – there will be some proportional increase in emissions. These emissions will then affect the global climate and cause damage elsewhere, which will in turn damage any forthcoming production (Mizutori, 2017). Computationally, the DICE model accounts for the accumulation of emissions in its climate module. Specifically, all emissions are converted to an equivalent parts per million of carbon dioxide (ppm CO2) and the rate at which an increase in carbon density can affect the average global temperature is derived from the standard radiative forcing equation to yield:

Equation 22: Temperature as a Function of Emissions

where E(0) is the initial density of CO2e and E(t) is the density under examination (Nordhaus, Apr. 2017). Incorporating temperature induced damages on Y(t) using the default constants from DICE yields the following system:

Equation 23: Predator-Prey Emissions Function

Equation 24: Predator-Prey Atmospheric Carbon Transfer Function

Equation 25: Predator-Prey Lower Ocean Carbon Transfer Function

Equation 26: Predator-Prey Gross Output Function

Where is the damage 1ºC has on the growth rate of Y(t) , =3% is the average annual growth rate of Y(T), and is the ratio of carbon emissions to gross output (Cooke, 2012). To clearly demonstrate the dynamics of this system, consider that the change in Y(t) is:

Equation 27: Predator-Prey Gross Output Function (equivalent form)

Which is to say that the increase in Y(t) is the annual growth rate, , minus the damage to that growth rate by climate change as a result of emissions, . A separable differential equation for Y(t) can be defined as:

Equation 28: Predator-Prey Differential Gross Output Function

with implicit solution:

Equation 29: Solution of Differential Predator-Prey Gross Output Function

Using the constants defined in Cooke 2012, it can be shown that growth becomes negative at T = 6ºC. In calculating the model with an initial Y(t) of $69.79 trillion over the course of 500 years, Y(t) grows to a maximum of $303 trillion after roughly 125 years, after which Y(t) collapses to $39.5 trillion and production remains lower than the initial for roughly 2000 years. Unsurprisingly the growth in emissions follows a similar pattern to that of Y(t) in that it reaches its peak alongside production, in roughly 125 years, and then collapses. Unlike Y(t), emissions do not recede to their initial amount likely due to the long-term carbon storage associated with the deep oceans. In summary, these calculations suggest that – with enough time – emissions induced climate change can have a negative effect on the global economy in a form that is similar to traditional predator-prey dynamics.

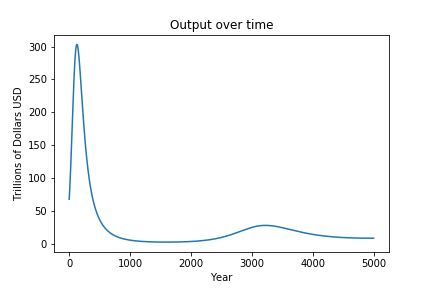
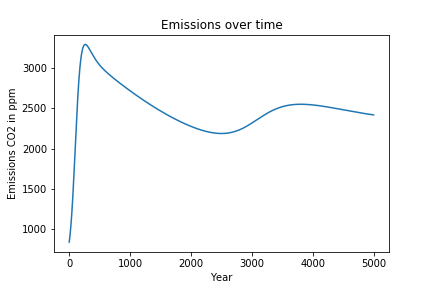


Figure 3:Emissions over time under Predator-Prey Dynamics

Figure 4:Output over time under Predator-Prey Dynamics

The computations and examples provided by Cooke 2012 demonstrate that the economics of climate change are more complex than what is afforded by the current dynamics in DICE. That said, there have been many advancements in the study of climate on the economy since this publication. In particular, the constants , , and can be updated to reflect the modern research, and affect the dynamics and computations of the model. The value that is least altered is . Where Cooke 2012 sites the World Bank average annual growth rate of Y(t) as 3% in 2012, with the economic advancements of the past decade a more reasonable value is =3.48% (O’Niell, 2023). The damage constant, , was the most understudied at the time, and is updated to =.008 (WebDICE, University of Chicago). Lastly the ratio of carbon emissions to Y(t), , becomes =.374 to coincide with recent economic and population booms (O’Niell, 2023). With these updated values, growth in Y(t) becomes negative at 3ºC and Y(t) grows to a maximum of $93 trillion after roughly 50 years, beyond which it collapses to 0 and production remains nonexistent. Again the growth of emissions follows a similar pattern, but instead reach a maximum of 2607 ppm CO2 and stabilize lower than in the original model.

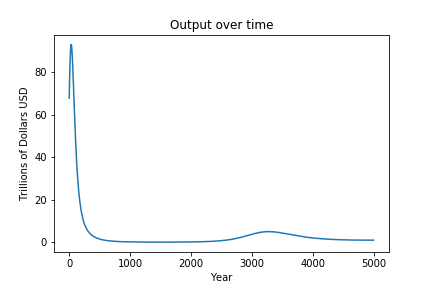
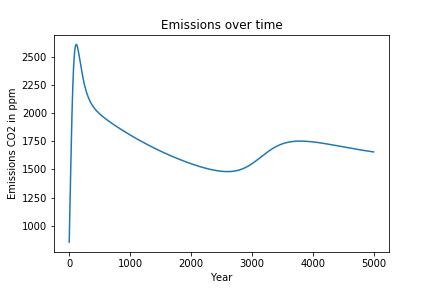


Figure 5: (Left) Emissions over time under Predator-prey with updated constants

Figure 6: (Right) Output over time under Predator-prey with updated constants

Implementing updated values for some of the parameters defining the relationship between climate and economy have accelerated the behaviors originally described in Cooke 2012. To that end, additional research into the economic impact on climate, has revealed that the constant parameters present in DICE are better described by functions (Keen, 2021). Damage to economic growth by climate change for example, is more dependent on the stage of the economy and is thus not adequately described by a single ratio. In particular, the damages associated with newly industrialized economies are greatly outweighed by the economic growth afforded by industrialization (Ritchie et. al., 2022). In such economies there is less incentive to implementing climate-friendly energy production as the initial cost is a significant barrier of entry (IEA, Global Energy Review), whereas industrialized economies have the available capital to alter their existing industries.

Economists have studied this phenomenon in great detail, but not much work has been done to implement these findings in climate-economy models (Keen, 2021). One reason is that the distribution of emissions by the level of advancement of an economy does not directly reflect that economies carbon footprint. Industrialized economies which are more reliant on manufacturing (called Secondary Economies), or services and retail (called Tertiary Economies) distribute much of their emissions to lower level economies (Center for Global Development). Where a Tertiary Economy may sell an article of clothing on the market, often the emissions released in creating that clothing and transporting it to market are not associated with that economy. To integrate these economic level based emissions relationships, an alternate damage function would be of the form:

Equation 30: Alternate Damage Function

With representing the portion of emissions from each type of economy (Primary, Secondary, or Tertiary) and , the rate of emissions growth generated for that economy.

Another impactful update the existing model is to implement a non-linear growth in Y(t). The assumption that the global economy can be adequately described by a linear function is useful and applicable in many situations, but cannot respond to unexpected events adequately. In particular, the recent Covid-19 pandemic had a drastic effect on the global economy that is not at all represented in a linear growth function (O’Niell, 2023), and the outbreak of war in Ukraine negatively impacted the economy for many months in 2022 (Wall Street Journal, 2022). Implementing variables to account for the wider effects of pandemics and wars would better describe modern economic growth. In such a model, economic growth would be defined by:

Equation 31: Alternate Economic Growth Function

Where is the damage to growth associated to war; is the rate at which wars occur with period ; is the damage to growth of Y(t) associated to natural disasters, and is the number of natural disasters occurring in year t.

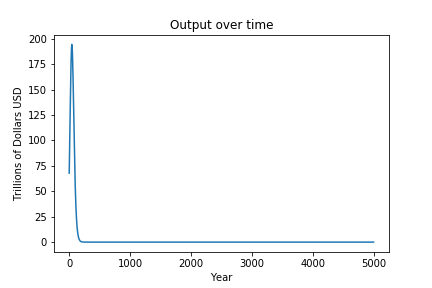
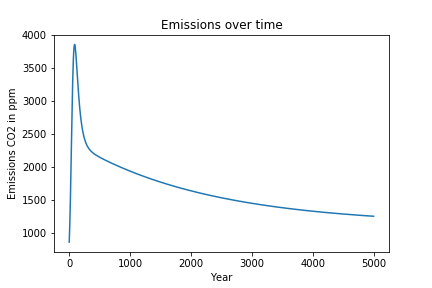


Figure 7: (Left) Emissions with Alternate Damage Function and Economic Growth

Figure 8: (Right) Output with Alternate Damage and Economic Growth

The addition of nonlinear economic damages and economic growth expectedly has the effect of accelerating much of the behavior displayed in the predator-prey model. The sharp increase in both emissions and output indicate that the negative effects emissions can have on the economy will not take effect until some tipping point, which agrees with what is thought by climate scientists (Kellett et. al., 2019). Interestingly, the long term effects indicate there will not be an economic resurgence like is displayed in the original model. This may be attributed to longer lasting negative effects of climate as a result of higher emissions. An alternate explanation may be that the impact of future wars or natural disasters is much higher. These increased damages are expected to occur more frequently as the climate continues to change (Gillingham, 2019). In turn, access to potable water, sufficient food supplies, and territory is expected to decline the next 50 years (Mizutori, 2019), and these tensions are believed to bring about more conflict, and by extension, hinder economic output.

# Shared Resource DICE 2016

Other possible areas of improvement come from implementing alternative competition dynamics between the various economic and geophysical modules. One such model would define a more traditional competitive relationship between the climate and economy. In this model, the climate and economy would be consuming a shared resource – the planet’s stock of natural resources – in which the economy would use those resources to generate capital, and the climate would use those resources to sequester carbon (United Nations Food and Agriculture Organization, 2020). To create such a model, a function for the amount of available resources at year t is defined:

Equation 32: Function for Sum of Natural Resources

Where S(t) is the amount of renewable (or sustainable) natural resources available at time t, US(t) is the amount of non-renewable (or unsustainable) natural resources available at time t, and LA(t) is the amount of undeveloped land at time t. Each resource function is then defined as:

Equation 33: Renewable Resource Function

Equation 34: Non-Renewable Resource Function

Equation 35: Undeveloped Land Function

Where is the amount of renewable energy produced in exajoules in year t (Renewable Electricity – Analysis, IEA) where ; is the rate at which resources are renewed (Renewable Electricity – Analysis, IEA); is the rate at which non-renewable resources are consumed in exajoules (Friedlingstein et. al., 2022); is the initial amount of non-renewable resources in exajoules (Richie et. al., 2022); is the amount of land developed in hectares in year t (Ritchie and Roser “Deforestation and Forest Loss” where , 2021; Food and Agriculture Organization of the United Nations, 2020) where ; and is the rate that land is undeveloped (Ritchie and Roser “Aforestation”, 2021).

The shared consumption of natural resources is then implemented in the equations for capital and emissions as:

Equation 36: Natural Resource Competition Capital Function

Equation 37: Natural Resource Competition Emissions Function

to reflect that the capital generated increases by consuming natural resources, that amount of consumed natural resources contribute to emissions, and any natural resources generated will sequester emissions. It follows the continuous approximation is given by:

Equation 38: Natural Resource Competition Differential Capital Function

which is a first order linear ordinary differential equation. Letting c1 = K(0), the general implicit solution is:

Equation 39: Natural Resource Competition Differential Capital Function Solution

As for emissions, the continuous equation is given by:

Equation 40: Natural Resource Competition Differential Emissions Function

which is again a first order linear ordinary differential equation. Letting c1 = E(0), the general implicit solution is:

Equation 41: Natural Resource Competition Differential Emissions Function Implicit Solution

where:

Equation 42

It follows from equation 40 that:

Equation 43: Equivalent Emissions Substitution Function

Substituting this into equation 41 gives:

Equation 44

Note that at time t, E(t-1) has been calculated and is thus constant. This allows for a simple separation of the integral into:

Equation 45

Now consider from equation 41 that:

Equation 46

which then allows for a substitution into equation 45 to yield:

Equation 47: Natural Resource Competition Differential Emissions Function Solution

where E(0) and E(t-1) are constants. This definition suggests that at any point in time, the next emissions value is most heavily influenced by the initial amount of emissions, E(0), and the most recent amount of emissions, E(t-1) which has been depreciated by the rate of emissions sequestering (1+s+l).

With these changes, the model computes emissions and output to rise at near-equal rates until the stock of non-renewable natural resources is fully consumed. Research indicates that non-renewable energy sources are expected to be fully exhausted within 150 years at the current rate of consumption and growth (Ring, 2022). The shared resource model accounts for this depletion and quantifies the impact as a slowdown in output, and at that time the available renewable energy may not meet demand if advancement in renewable energy production continues at the current rate (depicted in figure 12).

Interestingly, the impact of land development is much greater than that of energy, as a total loss of undeveloped land causes both emissions and output to collapse. Beyond this point the model indicates a swift rebound, but this is likely in error as a world devoid of natural resources is unlikely to support economic growth.

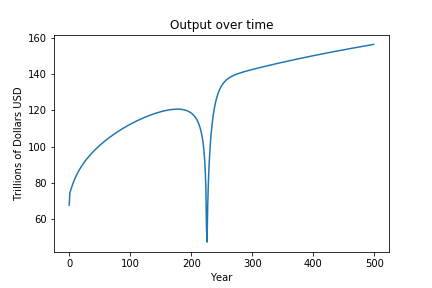
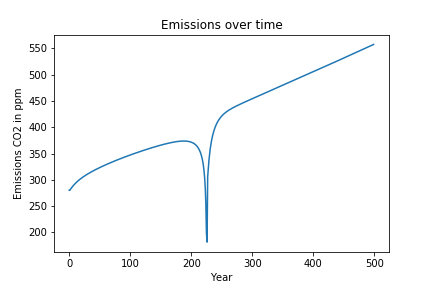


Figure 9: (Left) Emissions under resource competition

Figure 10: (Right) Output under resource competition

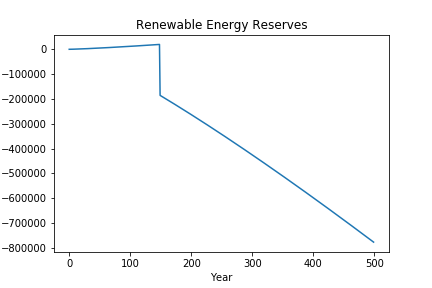
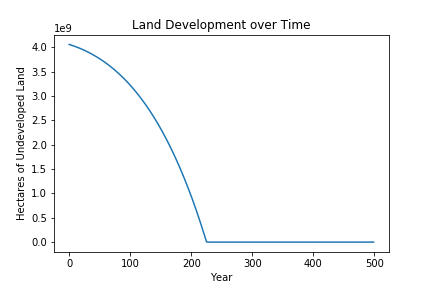


Figure 11: (Left) Development of land under resource competition

Figure 12: (Right) Production of Renewable Energy under resource competition

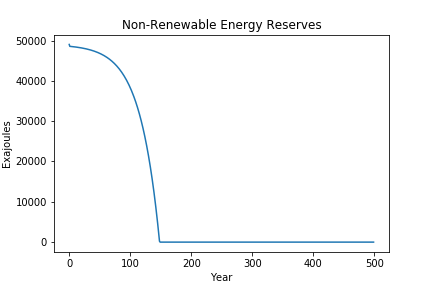


Figure 13: Consumption of Non-Renewable Resources under resource competition

# Legal Monopoly DICE 2016

There is much debate over how quickly climate mitigation strategies should be implemented and how they should be enforced. The current approach is mostly elective, with agreements such as the Paris Agreement defining caps on emissions and setting goals for renewable energy creation (UN Climate Press, 2022). Though progress has been made to meet the goals of the Paris Agreement, the United Nations Framework Convention on Climate Change (UNFCCC) is of the mind that the goals set by the accord are much further out than is desirable, and are calling for stricter policies to reach the proposed climate goals. To that end, the strictest possible policy would be akin to a legal monopoly. In this system, economic output is undisturbed when predetermined emissions targets are met, and there are penalties for noncompliance. The dynamics of such a system could vary greatly from other competitive models depending on the penalty. To begin, a function defining the climate goals is defined as:

Equation 48: Emissions Goal Function

where is the intended target of emissions reductions. For example, indicates that the intended emissions reductions for a given year are a 5% decrease from the previous year. This goal function will then be implemented alongside a penalty function:

Equation 49: Penalty Function

with being the intended economic penalty when emissions are over the limit. Building these functions into the existing DICE framework yields the following equation for gross output:

Equation 50: Natural Monopoly Gross Output Function

where:

Equation 51: Max function

Observe that if max{G(t),E(t)} is G(t), then the exponent on P(t) is 0, and thus P(t) = 1; so there is no penalty on output. Instead if max{G(t),E(t)} is E(t), the exponent is G(t) – E(t) and P(t) yields the desired penalty .

With these changes the model computes a system very similar to the standard DICE when the penalty on output is small. For an emissions target of =.1 and a penalty of , there is a noticeable decrease in output, but output continues to grow steadily even when emissions targets are not met. It follows that in such a system, is more flexible than strict of a goal. This behavior is entirely supported by the penalty, , as by increasing it the dynamics of the system vary greatly. With a penalty of , the system oscillates yearly. In one year the emissions targets are not met, and so there is a large penalty. In the following year emissions targets are met as a result of lower investment, and there is no penalty, so the economy once again experiences growth. Emissions follow a similar trend, in that, one year they are below the goal and the next above. These dynamics suggest that controlling an economy with a penalty for not meeting emissions targets may not have the intended effect of curbing overall emissions if the penalty is too high.

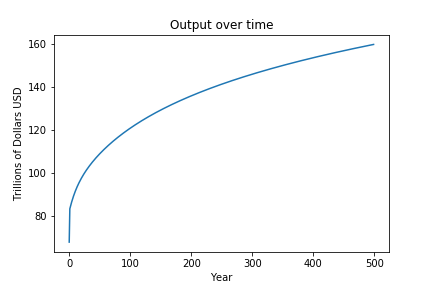
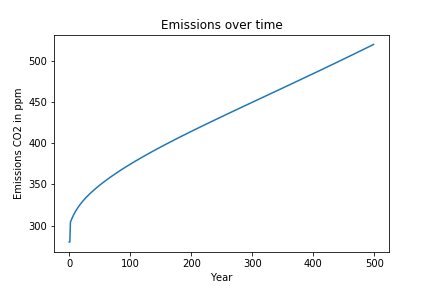


Figure 14: (Left) Emissions under Legal Monopoly

Figure 15: (Right) Output under Legal Monopoly

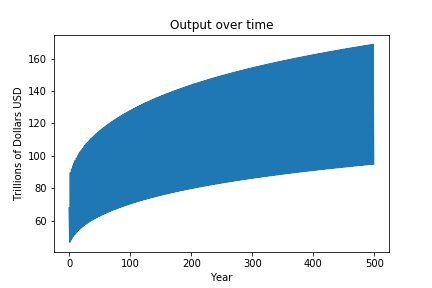
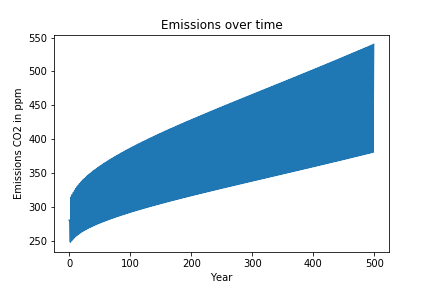


Figure 16: (Left) Alternate Emissions under Legal Monopoly

Figure 17: (Right) Alternate Output under Legal Monopoly

# Continuous Representations of Discrete Functions

Traditionally economic IAMs are defined discretely (Nordhaus, Feb. 2017), but that does not necessarily suggest this approach is optimal. Discrete models are convenient for their ease in programming, and are generally used to study the changes that take place in the modeled system – which is why they are commonly referred to as Difference Equations. That said, discrete models are not always the best approach for representing a system, in particular if time is an important factor. Discrete models calculate their values in static non-negative integer intervals of time. It follows that intermediate values are less accurate in discrete models, and that a larger interval necessitates more calculation time (Hundley, 2007). Many of the issues resulting from the limitations of discrete models can be rectified with continuous models.

As the name suggests, continuous models calculate their values with a continuous time variable. This allows for greater accuracy in calculating intermediate values, but includes the added benefit of being subject to alternate analytical techniques (Hundley, 2007). Converting a discrete time model to a continuous time model can be as simple as taking a derivative – provided that the functions are smooth. Another common method is to approximate the derivative of a function that operates over large time scales and solving the resulting differential equation by redefining it as follows (Hritonenko and Yatsenko, 2010):

Equation 52: Differential Approximation

Using the Cooke predator-prey model as an example, a continuous function for emissions is approximated by matching the form. To do so define:

Equation 53: Condensed Biosphere Carbon Transfer Function

Equation 54: Equivalent Predator-Prey Emissions Function

which has the differential equation approximation:

Equation 55: Differential Predator-Prey Emissions Function

with implicit solution:

Equation 56: Solution to Differential Predator-prey Emissions Function

This definition provides further clarity on the relationship between emissions and output in the predator-prey model. Observe the solution at t = 0 is E(0) as expected. An interesting observation is that the initial emissions term is multiplied by a negative exponential. It follows that the initial amount of emissions become less important as time progresses, and rather the emissions generated by the biosphere and output functions primarily drive growth. It is important to note that the only way emissions can significantly reduce over time is if the output term decreases. This suggests that growth in output comes at the cost of additional emissions, which again demonstrates the predator prey relationship between carbon emissions and economic output that has been implemented in the model.

Implementing a continuous time version of DICE is unlikely to change the overarching results of the model, as the intent is to define an equivalent differential equation (or at least as equivalent of an approximation as possible). That said, there can be benefit in doing so. As previously demonstrated, the continuous time definition of a function can provide further clarity in defining the relationships between variables, but this form now allows for better examination of the function as a whole (Babuska and Oden, 2004). It is well known that taking the integral any function calculates the area under the curve within the chosen bounds, but an alternate interpretation is that the integral of a function is the average value of the function multiplied by the length of the bounds (Hughes-Hallett et. al., 2021). It follows that taking the average value via the integral of the continuous emissions equation would reveal what the total amount of emissions were over the chosen time period, and the average can then be easily found. Though for small periods of time the average can be calculated more traditionally (by summing the values and dividing by the number of values), for longer periods of time taking the integral is far less computationally expensive (Hudley, 2007).

Though continuous versions of discrete equations have their advantages, there is not always a benefit in doing so. Deriving a continuous model from a discrete equation is highly dependent upon the length of the time interval present in the discrete version (Paez, 2008, Tung 2007). Whereas the discrete model only uses the most recent iteration of values to calculate its next values, continuous models explicitly take the elapsed time since the previous observation into account when calculating the current state (Loossens et. al., 2021). Using equations 23 and 51, the differences in values and are exemplified with the following: suppose Y(t) and M\*(t) remain constant for 50 years. The continuous expression then reduces to:

Equation 57: Continuous Emissions Equation Over 50 Years

Compared to the discrete version, the equations yield a percent error of, at maximum, 4.46% with the majority of the error coming from the first 30 years where the continuous equation is underestimating the growth in emissions. That said, computing the integral for different lengths of time in this fashion will yield alternate equations. For example, if Y(t) and M(t) remain constant for 200 years, the continuous expression becomes:

Equation 58: Continuous Emissions Equation Over 200 years

and the percent error now achieves a maximum of 8.37% around 100 years in by again underperforming the growth of the discrete function. This overall suggests the continuous emissions equation may underestimate the discrete equation to account for the growth in output and carbon density being more dynamic and that intermediary values are likely less impactful in the discrete equations than they should be.

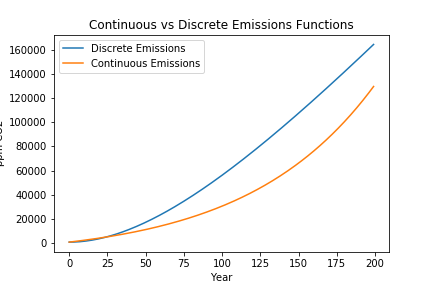
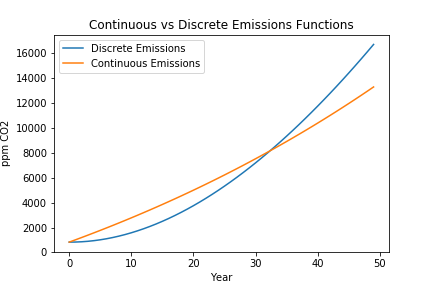


Figure 18: (Left) Discrete vs Continuous Emissions Functions Over 50 years

Figure 19: (Right) Discrete vs Continuous Emissions Functions Over 200 years

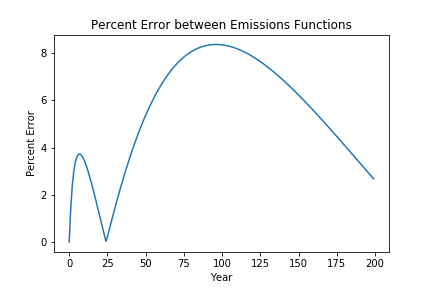
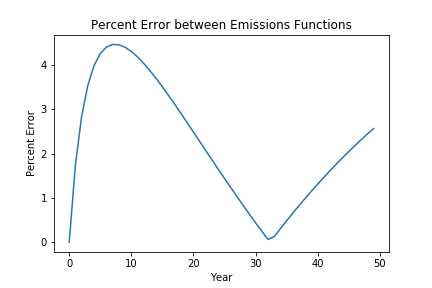


Figure 20: (Left) Discrete vs Continuous Percent Error Over 50 years

Figure 21: (Right) Discrete vs Continuous Percent Error Over 200 Years

# Conclusion

As demonstrated throughout this paper, it is likely that the existing mechanics of the DICE model do not accurately reflect the relationship between the global climate and economy. The original dynamics in the present model suggest the impact climate change can have on economic output will plateau as time goes on, and that even under a drastically different global climate where average temperature has increased by 20 ºC, capital stocks are only reduced by half. In implementing simple predator-prey dynamics between output and emissions as suggested in Cooke 2012, the DICE model suggests both emissions and economic output will peak in roughly 125 years, after which there will be significant and sustained collapse for both. Expanding upon the Cooke model by updating values for economic growth and economic damages due to climate change associated events, the model then accelerates the new behavior such that the predetermined peak in emissions and output occur in roughly 60 years, and the following collapse is both more drastic and sustained over time.

While predator prey dynamics may be a more accurate representation of the climate-economy relationship than those of the original DICE model, shared resource competition is more reflective of real-world economics than either (Grubb et. al., 2021). With such an implementation, economic output and emissions are expected to grow logarithmically until the supply of natural resources of the planet are depleted. At this point, output and emissions both plummet as without resources, it is very likely existing societies will collapse (Economic Activity Indicators, International Monetary Fund). Beyond this point the model computes a rebound, which is either an error in implementation or suggests that development of renewable resources will advance to the point that the global economy can recover if all efforts are devoted to recovery (Renewables – Global Energy Review 2021, IEA).

More than examining the interdependent relationship between the global climate and economy, implementing financial penalties to simulate emissions agreements (or carbon taxes) can result in a system analogous to legal monopolies. Under such a structure, both emissions and economic output continue to grow indefinitely – though at a much slower rate than in the standard DICE or any other examined model. To that end, larger penalties on output cause the system to fluctuate drastically while continuing to increase over time. That said, such a sharp penalty on economic output for one year would more likely crush an economy (Grubb et. al., 2021) than allow it to rebound the following year, and suggests further study is needed.

In summary, IAMs such as DICE can be useful tools in approximating the effects various policies can have on the outcomes of the global economy; but providing a definitive prediction of the future is impossible. Global finance is highly variable and dependent on many relationships that are difficult to implement in a model: such as political events, natural disasters, or technological advancement. Further, the relationship between the global climate and economy is likely more complex than in any of the proposed models, and even a combination of the examined models may not adequately reflect the interconnection between the two (Grubb et. al. 2021). To that end, there is room for further work on creating realistic models of the global economy integrated with the effects of climate change.

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# Appendix

## ﻿DICE Predator Prey Code

﻿#!/usr/bin/env python2

# -\*- coding: utf-8 -\*-

"""

Created on Fri Apr 14 13:44:43 2023

@author: jarrettvalenti

"""

#Imports

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

#Initial Values

T=5000#num years

K\_0=135#67.79 #inital gwp

N\_0=6838#initial population

A\_0=3.8#initial tech factor

I\_0=16.38#initial investment

alpha=0.009 #damage to Y

beta=0.0485 #increase to Y

gamma=0.3

delta=0.1

eps=.1 #emissions/Y

Mat\_0=588 #atmosphere initial co2

Muo\_0=1350 #upper ocean initial co2

Mlo\_0=10010 #inital lower ocean co2

E\_0=280 #historical value of ghg

E\_1=830 #inital value of ghg

Eland\_0=3.3#initial Eland co2

cs=3.6 #climate sensitivity

T\_0=np.log(2)\*np.log(E\_1/E\_0)\*cs #initial avg temp

#Arrays to hold values

Y=[]

A=[]

K=[]

I=[]

N=[]

E=[]

Mat=[]

Mlo=[]

Temp = []

for t in range(0,T):

if(t==0):

y= 67.79

a= A\_0

k= K\_0

i= I\_0

n= N\_0

e=0.988\*E\_1+0.0047\*(Mat\_0+Muo\_0)+eps\*y

mat=0.9948\*(Mat\_0+Muo\_0)+.012\*E\_1+.0001\*Mlo\_0

mlo=0.9999\*Mlo\_0 + 0.0005\*(Mat\_0+Muo\_0)

temp=T\_0

else:

y = Y[t-1] + Y[t-1]\*beta - alpha\*Temp[t-1]\*Y[t-1]

a = 3.8+.079\*t

k= (1-delta)\*K[t-1]+I[t-1]

i= 16.38\*t

n= 6838 + 1.9\*t

e= 0.988\*E[t-1]+0.0047\*(Mat[t-1])+eps\*Y[t-1]

mat= 0.9948\*(Mat[t-1])+.012\*E[t-1]+.0001\*Mlo[t-1]

mlo=0.9999\*Mlo[t-1] + 0.0005\*(Mat[t-1])

temp=np.log(2)\*np.log(E[t-1]/E\_0)\*cs

Y.append(y)

A.append(a)

K.append(k)

I.append(i)

N.append(n)

E.append(e)

Mat.append(mat)

Mlo.append(mlo)

Temp.append(temp)

#output data

data = pd.DataFrame({'Emissions':E, 'Output':Y, 'Capital':K, 'delta-Temp':Temp})

data.to\_csv('DICE\_L\_V.csv')

#plotting output

plt.figure(1)

plt.plot(Y[0:500])

plt.title("Output over time")

plt.xlabel("Year")

plt.ylabel("Trillions of Dollars USD")

plt.savefig("DICE L\_V\_Y\_1.png")

#plotting emissions

plt.figure(2)

plt.plot(E)

plt.title("Emissions over time")

plt.xlabel("Year")

plt.ylabel("Emissions CO2 in ppm")

plt.savefig("DICE L\_V\_E\_1.png")

## DICE Predator Prey (updated) Code﻿

#!/usr/bin/env python2

# -\*- coding: utf-8 -\*-

"""

Created on Sat Apr 15 16:14:36 2023

@author: jarrettvalenti

"""

#Imports

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

#Initial Values

T=5000#num years

K\_0=135#67.79 #inital gwp

N\_0=6838#initial population

A\_0=3.8#initial tech factor

I\_0=16.38#initial investment

alpha=0.009 - 0.005 + 0.008#damage to Y

beta=0.0485 -0.03 + 0.0348 #increase to Y

gamma=0.3

delta=0.1

eps=0.1 -0.1 + 0.374#emissions/Y

Mat\_0=588 #atmosphere initial co2

Muo\_0=1350 #upper ocean initial co2

Mlo\_0=10010 #inital lower ocean co2

E\_0=280 #historical value of ghg

E\_1=830 #inital value of ghg

Eland\_0=3.3#initial Eland co2

cs=3.6 #climate sensitivity

T\_0=np.log(2)\*np.log(E\_1/E\_0)\*cs #initial avg temp

#Arrays to hold values

Y=[]

A=[]

K=[]

I=[]

N=[]

E=[]

Mat=[]

Mlo=[]

Temp = []

for t in range(0,T):

if(t==0):

y= 67.79

a= A\_0

k= K\_0

i= I\_0

n= N\_0

e=0.988\*E\_1+0.0047\*(Mat\_0+Muo\_0)+eps\*y

mat=0.9948\*(Mat\_0+Muo\_0)+.012\*E\_1+.0001\*Mlo\_0

mlo=0.9999\*Mlo\_0 + 0.0005\*(Mat\_0+Muo\_0)

temp=T\_0

else:

y = Y[t-1] + Y[t-1]\*beta - alpha\*Temp[t-1]\*Y[t-1]

a = 3.8+.079\*t

k= (1-delta)\*K[t-1]+I[t-1]

i= 16.38\*t

n= 6838 + 1.9\*t

e= 0.988\*E[t-1]+0.0047\*(Mat[t-1])+eps\*Y[t-1]

mat= 0.9948\*(Mat[t-1])+.012\*E[t-1]+.0001\*Mlo[t-1]

mlo=0.9999\*Mlo[t-1] + 0.0005\*(Mat[t-1])

temp=np.log(2)\*np.log(E[t-1]/E\_0)\*cs

Y.append(y)

A.append(a)

K.append(k)

I.append(i)

N.append(n)

E.append(e)

Mat.append(mat)

Mlo.append(mlo)

Temp.append(temp)

#output data

data = pd.DataFrame({'Emissions':E, 'Output':Y, 'Capital':K, 'delta-Temp':Temp})

data.to\_csv('DICE\_L\_V.csv')

#plotting output

plt.figure(1)

plt.plot(Y[0:500])

plt.title("Output over time")

plt.xlabel("Year")

plt.ylabel("Trillions of Dollars USD")

plt.savefig("DICE L\_V\_Y\_2.png")

#plotting emissions

plt.figure(2)

plt.plot(E)

plt.title("Emissions over time")

plt.xlabel("Year")

plt.ylabel("Emissions CO2 in ppm")

plt.savefig("DICE L\_V\_E\_2.png")

## DICE Shared Resource Code

﻿#!/usr/bin/env python2

# -\*- coding: utf-8 -\*-

"""

Created on Sat Apr 15 16:21:06 2023

@author: jarrettvalenti

"""

#Imports

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

#economy Initial Values

T=5000#num years

K\_0=135#67.79 #inital gwp

N\_0=6838#initial population

A\_0=3.8#initial tech factor

I\_0=16.38#initial investment

Y\_0=67.78

gamma=0.3

delta=0.1

Q\_0=67.79

beta=0.0485

#climate initial vals pulled from DICE

psi\_1=0.00267

psi\_2=2

Tat\_0=0.8

Tlo\_0=0.0068

z\_1=0.098

z\_2=1.31

z\_3=0.088

z\_4=0.025

O\_0= (1-(1/(1+psi\_1\*Tat\_0 + psi\_2\*Tat\_0\*\*(2))))

sigma\_0=0.549

sigma=.01#change to sigma

mu\_0=.001\*sigma\_0

theta\_1=sigma\_0/2800

theta\_2=2.8

V\_0=theta\_1\*(mu\_0\*\*theta\_2)

Mat\_0=588 #atmosphere initial co2

Muo\_0=1350 #upper ocean initial co2

Mlo\_0=10010 #inital lower ocean co2

phi\_at\_at=0.912

phi\_at\_uo=0.088

phi\_uo\_at=0.0383

phi\_uo\_uo=0.9592

phi\_uo\_lo=0.0025

phi\_lo\_uo=0.0003375

phi\_lo\_lo=0.9996625

E\_0=280 #historical value of ghg

E\_1=830 #inital value of ghg

Eland\_0=3.3#initial Eland co2

eps=.1 #emissions/Y

fex=0.7

#Shared resources initial vals

US\_0=10596+31471+6959#reserves in exajoules as of 2020

us\_var=131+151+174 #consumption in exajoules 2020

S\_0= 77.5#around 17% of energy generated each year is renewable as of 2020

LA\_0=4060000000#hectares of undeveloped land

NR\_0=S\_0+LA\_0+US\_0#sum

s\_var=.05#growth of renewables

S\_var= 40#current consumption of renewables in exajoules

l\_var=0.0015#rate of aforestation

L\_var=10000000#hectares consumed per year as of 2010-2020

dl=.13#ratio of econ attributed to land use

#Arrays to hold values

Y=[]

A=[]

K=[]

I=[]

N=[]

E=[]

Eland=[]

Q=[]

Mat=[]

Muo=[]

Mlo=[]

Tat = []

Tlo=[]

O=[]#damages

V=[]#abatement

F=[]#forcings

#shared resources arrays

NR=[]

S=[]

US=[]

LA=[]

for t in range(0,T):

if(t==0):

#nat resc

nr=NR\_0

s=max(S\_0\*(1 + s\_var) - S\_var\*(1 + dl),0)

us=US\_0

la=LA\_0

#econ

a= A\_0

k= K\_0

i= I\_0

n= N\_0

y= Y\_0

#clim

o=O\_0

v=V\_0

sig = sigma\_0

mu= mu\_0

eland=Eland\_0

e=E\_0

mat=Mat\_0

muo=Muo\_0

mlo=Mlo\_0

f=5.33\*np.log((mat)/Mat\_0)/np.log(2) + fex

tat=Tat\_0

tlo=Tlo\_0

q=Q\_0

else:

#nat resc

us = max(US\_0 - us\_var\*(1+beta)\*\*(t/1.5),0)

if(us==0):#when out of resources demand will shift to renewables

s = min(S\_0\*(1 + s\_var)\*\*(np.log(t)) - (S\_var+us\_var)\*(1 + beta)\*\*(25\*np.log(t)),#75\*s\_var

1.2\*(S\_var+us\_var)\*(1 + beta)\*\*(25\*np.log(t)))

else:

s = S\_0\*(1 + s\_var)\*\*(25\*np.log(t)) - S\_var\*(1 + beta)\*\*(25\*np.log(t))

la = max((1 + l\_var)\*LA[t-1] - (L\_var\*(1+beta\*dl)\*\*(t)),0)

nr= (s + la+ us)

#econ

a = np.log(3.8 + 0.079\*t) +2.5

i= 16.38\*(1.01)\*\*np.log((t))

k= (nr/NR[t-1] - delta)\*K[t-1] + (i)

n= 6838 + 1.9\*t

y = (a\*(k\*\*gamma)\*(np.log(n\*\*(1-gamma))))-30

#climate

o=(1-(1/(1+psi\_1\*Tat[t-1]+psi\_2\*Tat[t-1]\*\*(2))))

sig = sigma\_0+1.1\*\*(sigma\*t)

mu = 0.001\*sig

eland = max(Eland[t-1] + 0.2\*(1-la/LA[0]),0)

e = sig\*(1 - mu)\*y\*nr/NR[t-1] + eland - (s\_var + l\_var)\*E[t-1] + 176

mat = phi\_at\_at\*Mat[t-1] + phi\_uo\_at\*Muo[t-1]

muo = phi\_at\_uo\*Mat[t-1] + phi\_uo\_uo\*Muo[t-1] + phi\_lo\_uo\*Mlo[t-1]

mlo = phi\_uo\_lo\*Muo[t-1] + phi\_lo\_lo\*Mlo[t-1]

f = 5.33\*(np.log(mat/Mat\_0)/np.log(2)) + fex

tat = Tat[t-1] + z\_1\*(f - z\_2\*Tat[t-1] - z\_3\*(Tat[t-1]-Tlo[t-1]))

tlo = Tlo[t-1] + z\_4\*(Tat[t-1]-Tlo[t-1])

q = o\*(1 - v)\*y + 26

Y.append(y)

A.append(a)

K.append(k)

I.append(i)

N.append(n)

E.append(e)

Eland.append(eland)

Mat.append(mat)

Muo.append(muo)

Mlo.append(mlo)

Tat.append(tat)

Tlo.append(tlo)

O.append(o)

V.append(v)

Q.append(q)

F.append(f)

S.append(s)

US.append(us)

LA.append(la)

NR.append(nr)

#output data

data = pd.DataFrame({'Emissions':E, 'Output':Y, 'Capital':K, "Temperature":Tat})

data.to\_csv('DICE\_S\_R.csv')

#plotting output

plt.figure(1)

plt.plot(Y[0:500])

plt.title("Output over time")

plt.xlabel("Year")

plt.ylabel("Trillions of Dollars USD")

plt.savefig("DICE S\_R\_Y.png")

#plotting emissions

plt.figure(2)

plt.plot(E[0:500])

plt.title("Emissions over time")

plt.xlabel("Year")

plt.ylabel("Emissions CO2 in ppm")

plt.savefig("DICE S\_R\_E.png")

#plotting resourcess

plt.figure(3)

plt.plot(NR[0:500])

plt.title("Natural Resources over time")

plt.xlabel("Year")

plt.ylabel("Exajoules")

plt.savefig("DICE S\_R\_NR")

plt.figure(6)

plt.plot(S[0:500])

plt.title("Renewable Energy Reserves")

plt.ylabel("Exajoules")

plt.xlabel("Year")

plt.savefig("DICE\_S\_R\_S")

plt.figure(4)

plt.plot(US[0:500])

plt.title("Non-Renewable Energy Reserves")

plt.ylabel("Exajoules")

plt.xlabel("Year")

plt.savefig("DICE\_S\_R\_US")

plt.figure(5)

plt.plot(LA[0:500])

plt.title("Land Development over Time")

plt.ylabel("Hectares of Undeveloped Land")

plt.xlabel("Year")

plt.savefig("DICE\_S\_R\_LA")

## DICE Legal Monopoly Code

#!/usr/bin/env python2

# -\*- coding: utf-8 -\*-

"""

Created on Mon Apr 17 20:08:12 2023

@author: jarrettvalenti

"""

#Imports

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

#economy Initial Values

T=5000#num years

K\_0=135#67.79 #inital gwp

N\_0=6838#initial population

A\_0=3.8#initial tech factor

I\_0=16.38#initial investment

Y\_0=67.78

gamma=0.3

delta=0.1

Q\_0=67.79

beta=0.0485

#climate initial vals pulled from DICE

psi\_1=0.00267

psi\_2=2

Tat\_0=0.8

Tlo\_0=0.0068

z\_1=0.098

z\_2=1.31

z\_3=0.088

z\_4=0.025

O\_0= (1-(1/(1+psi\_1\*Tat\_0 + psi\_2\*Tat\_0\*\*(2))))

sigma\_0=0.549

sigma=.01#change to sigma

mu\_0=.001\*sigma\_0

theta\_1=sigma\_0/2800

theta\_2=2.8

V\_0=theta\_1\*(mu\_0\*\*theta\_2)

Mat\_0=588 #atmosphere initial co2

Muo\_0=1350 #upper ocean initial co2

Mlo\_0=10010 #inital lower ocean co2

phi\_at\_at=0.912

phi\_at\_uo=0.088

phi\_uo\_at=0.0383

phi\_uo\_uo=0.9592

phi\_uo\_lo=0.0025

phi\_lo\_uo=0.0003375

phi\_lo\_lo=0.9996625

E\_0=280 #historical value of ghg

E\_1=830 #inital value of ghg

Eland\_0=3.3#initial Eland co2

eps=.1 #emissions/Y

fex=0.7

#Arrays to hold values

Y=[]

A=[]

K=[]

I=[]

N=[]

E=[]

Eland=[]

Q=[]

Mat=[]

Muo=[]

Mlo=[]

Tat = []

Tlo=[]

O=[]#damages

V=[]#abatement

F=[]#forcings

#Monopoly Vars

G=[]

P=[]

a\_star=.4

for t in range(0,T):

if(t==0):

#econ

a= A\_0

k= K\_0

i= I\_0

n= N\_0

y= Y\_0#A\_0\*(K\_0\*\*gamma)\*(N\_0\*\*(1-gamma))

#clim

o=O\_0

v=V\_0

sig = sigma\_0

mu= mu\_0

eland=Eland\_0

e=E\_0

mat=Mat\_0

muo=Muo\_0

mlo=Mlo\_0

f=5.33\*np.log((mat)/Mat\_0)/np.log(2) + fex

tat=Tat\_0

tlo=Tlo\_0

q=Q\_0

#nat mon

else:

#climate

o=(1-(1/(1+psi\_1\*Tat[t-1]+psi\_2\*Tat[t-1]\*\*(2))))

sig = sigma\_0+1.1\*\*(sigma\*t)

mu = 0.001\*sig

eland = max(Eland\_0 + 0.2\*(np.log(t)),0)

e = sig\*(1 - mu)\*Y[t-1] + eland +172

mat = phi\_at\_at\*Mat[t-1] + phi\_uo\_at\*Muo[t-1]

muo = phi\_at\_uo\*Mat[t-1] + phi\_uo\_uo\*Muo[t-1] + phi\_lo\_uo\*Mlo[t-1]

mlo = phi\_uo\_lo\*Muo[t-1] + phi\_lo\_lo\*Mlo[t-1]

f = 5.33\*(np.log(mat/Mat\_0)/np.log(2)) + fex

tat = Tat[t-1] + z\_1\*(f - z\_2\*Tat[t-1] - z\_3\*(Tat[t-1]-Tlo[t-1]))

tlo = Tlo[t-1] + z\_4\*(Tat[t-1]-Tlo[t-1])

q = o\*(1 - v)\*y

#nat mon

g=.9\*E[t-1]

if(max(g,e)==e):

p=a\_star

else:

p=0

#econ

a = np.log(3.8 + 0.079\*t) +2.5

i= 16.38\*(1.01)\*\*np.log((t))

k= (1 - delta)\*K[t-1] + (i)

n= 6838 + 1.9\*t

y = (a\*(k\*\*gamma)\*(np.log(n\*\*(1-gamma)))\*(1-p))-16

G.append(g)

P.append(p)

Y.append(y)

A.append(a)

K.append(k)

I.append(i)

N.append(n)

E.append(e)

Eland.append(eland)

Mat.append(mat)

Muo.append(muo)

Mlo.append(mlo)

Tat.append(tat)

Tlo.append(tlo)

O.append(o)

V.append(v)

Q.append(q)

F.append(f)

data = pd.DataFrame({'Emissions':E, 'Output':Y, 'Capital':K, "Temperature":Tat})

data.to\_csv('DICE\_L\_M.csv')

#plotting output

plt.figure(1)

plt.plot(Y[0:500])

plt.title("Output over time")

plt.xlabel("Year")

plt.ylabel("Trillions of Dollars USD")

plt.savefig("DICE N\_M\_Y\_1.png")

#plotting emissions

plt.figure(2)

plt.plot(E)

plt.title("Emissions over time")

plt.xlabel("Year")

plt.ylabel("Emissions CO2 in ppm")

plt.savefig("DICE N\_M\_E\_1.png")

#run with a\_star=.4

plt.figure(3)

plt.plot(Y[0:500])

plt.title("Output over time")

plt.xlabel("Year")

plt.ylabel("Trillions of Dollars USD")

plt.savefig("DICE N\_M\_Y\_2.png")

plt.figure(4)

plt.plot(E)

plt.title("Emissions over time")

plt.xlabel("Year")

plt.ylabel("Emissions CO2 in ppm")

plt.savefig("DICE N\_M\_E\_2.png")

## Continuous vs. Discrete Equations Code

﻿

#!/usr/bin/env python2

# -\*- coding: utf-8 -\*-

"""

Created on Sat Apr 29 20:51:32 2023

@author: jarrettvalenti

"""

import numpy as np

import matplotlib.pyplot as plt

T = 200

E\_d = []

E\_c = []

E\_0=830

M= 588+1350

Y= 67.78

for i in range(T):

if i==0:

e\_d = E\_0

e\_c = E\_0\*np.e\*\*(-0.012\*i) + 13095.4\*np.e\*\*(0.012\*i)

else:

e\_d = .988\*E\_d[i-1]+0.0047\*M\*i + 0.1\*Y\*i

e\_c = E\_0\*np.e\*\*(-0.012\*i) + 13095.4\*np.e\*\*(0.012\*i)

E\_d.append(e\_d)

E\_c.append(e\_c)

for e in range(len(E\_c)):

E\_c[e]-=13095.4

plt.figure(1)

plt.plot(E\_d)

plt.plot(E\_c)

plt.title("Continuous vs Discrete Emissions Functions")

plt.xlabel("Year")

plt.ylabel("ppm CO2")

plt.legend(["Discrete Emissions", "Continuous Emissions"])

plt.savefig("cont\_disc\_emissions\_200.png")

pct=[]

for i in range(T):

pct.append(np.abs(E\_d[i]-E\_c[i])/(E\_c[i]) \* 100)

plt.figure(2)

plt.plot(pct)

plt.title("Percent Error between Emissions Functions")

plt.xlabel("Year")

plt.ylabel("Percent Error")

plt.savefig("pcte\_disc\_cont\_emissions\_200.png")